

Next week last class fuck! (exam probably at end of this month)

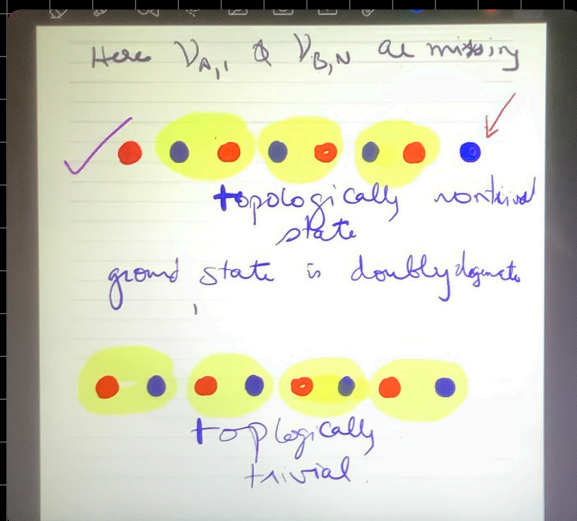
Topological Superconductivity

Majorana fermions in condensed matter

Easiest realizatⁿ in a Quantum vortex in a $p+ip$ superconductor
→ expt = 2/3 years earlier

↳ Kitaev's toy model :- Spinless p -wave SC
excitatⁿ on the boundary = linear combinatⁿ of fermionic operators.

Here $\psi_{A,1}$ & $\psi_{B,N}$ are missing



SOC + Superconductor

⇒ Primary → qubits that have some topological protection.

↓
@ Microsoft, @ Google, @ IBM

Weyl Semi-metals:-

→ what are weyl "particles/excitations" from high en. physics?

→ "Bulk itself is conducting"

→ what are the interesting properties of weyl particles

— Chiral anomaly

→ how does one realize a weyl hamiltonian in cond matter systems

→ Role of symmetry

→ Dimensions = odd (1 or 3)

→ Berry Curvature hotspots

→ Surface states/
fermi arcs

→ expt tests

→ ARPES

→ transport

→ Majorana with Weyl

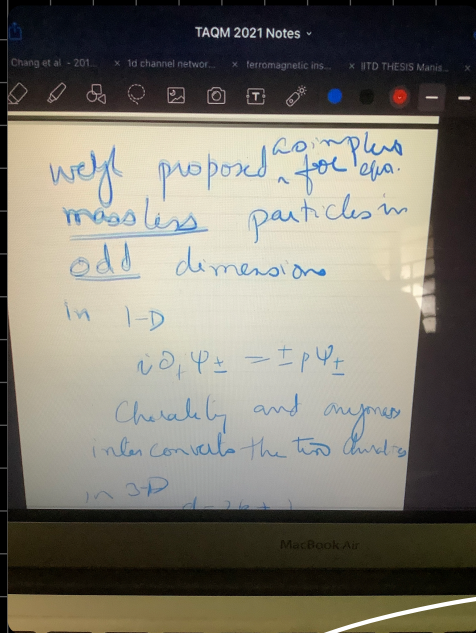
→ "Topological" weyl Superconductors

VANDERBILT

References
RMP
Vanderbilt
book

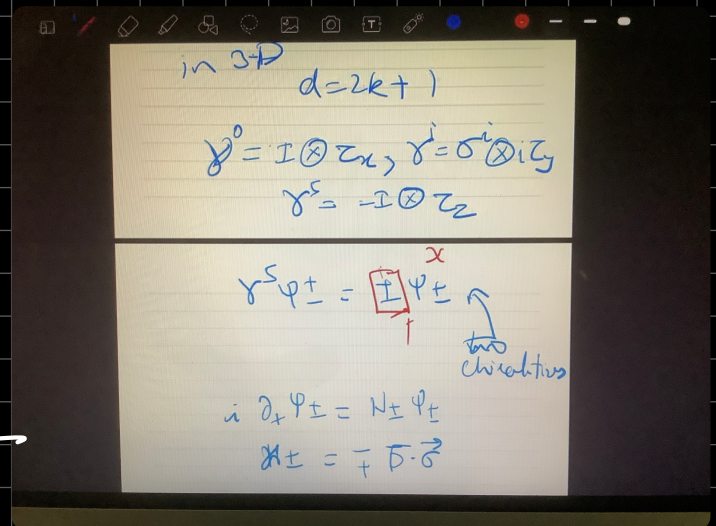
* Weyl from high en. physics.

Dirac eqn $\Rightarrow$



Chirality of the system

in 3D



two chiralities $\rightarrow \pm 1$ $\gamma^5 \psi_{\pm} = \pm \psi_{\pm}$

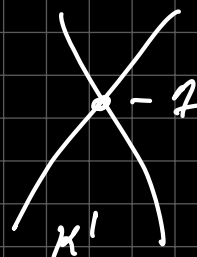
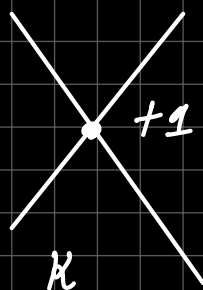
no real particles are identified as Weyl fermions.

Weyl fermions propagate parallel or antiparallel to their spin.

Chirality

2 of them come in pairs

$\Rightarrow$ linear dispersion in 3 dimensions



$\Rightarrow$ might seem like Graphene

$\Rightarrow$ "effective low energy hamiltonian" $\rightarrow$ graphene linearly $\sim 1 \text{ eV}$

Chiral Anomaly

large en. scale

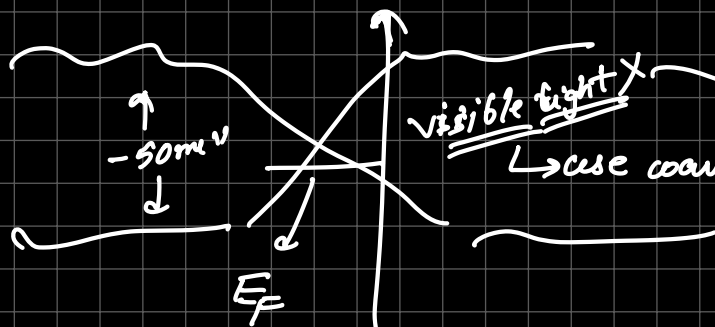
but for weyl, $\exists$ a smaller

(~ 50 meV) over which it's linear.

"system specific"

• this physics can be /
cannot be probed
depending on

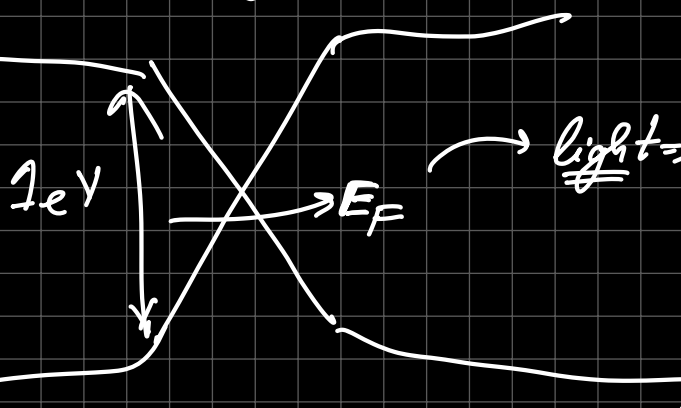
• $\exists$ an energy scale over which it's
an accurate description of reality



will easily probe the
large en. physics

$\sim k_B T \rightarrow$ low enough temperature

• graphene



drac
light $\rightarrow$ scattering by e & photon

"not a description of all quasiparticles at all energies."

Quasiparticles
"emergent excitation
of a system."

Chiral Anomaly

$\partial_\mu j^\mu = 0$ & $\partial_\mu j^\mu_5 = 0$ $\rightarrow$ conversion b/w right handed
to left handed

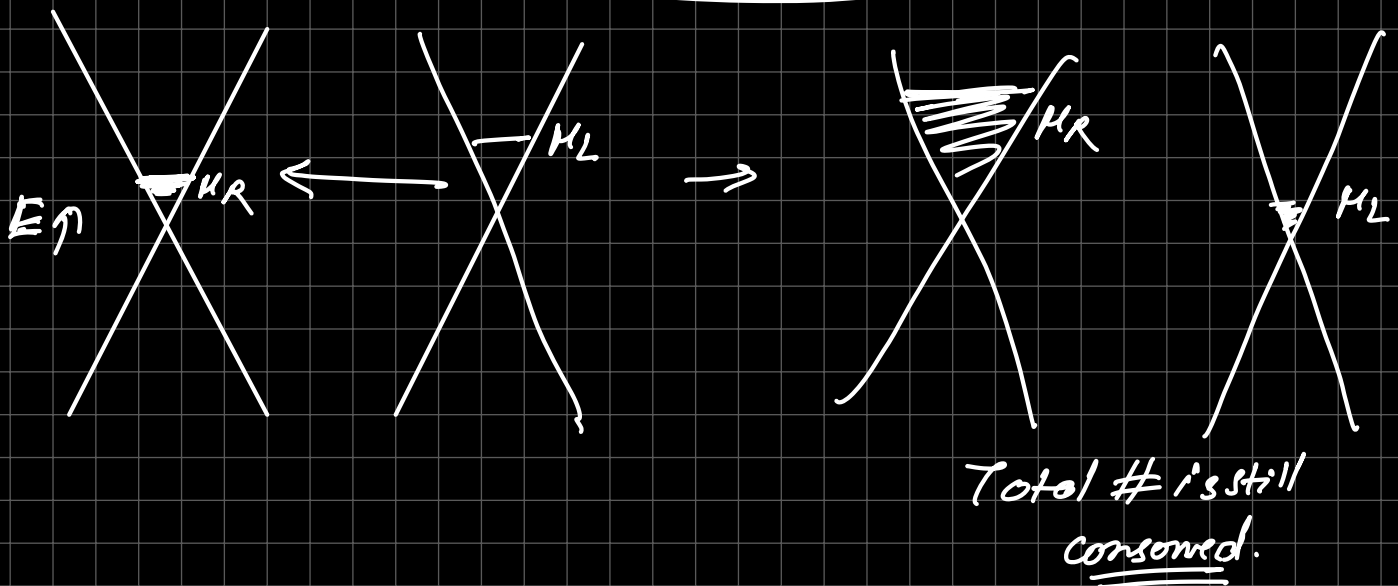
$\hookrightarrow$ independently # of left or right handed
weyl fermions aren't conserved.

$\partial_{\mu} j^{\mu 5} \neq 0 \Rightarrow$ cons. of each species isn't possible
 $\rightarrow$ conv. b/w right handed to left handed
weyl fermions.

"Total # of
 spin up spin down
cons."

$$\partial_t (n_R + n_L) = 0$$

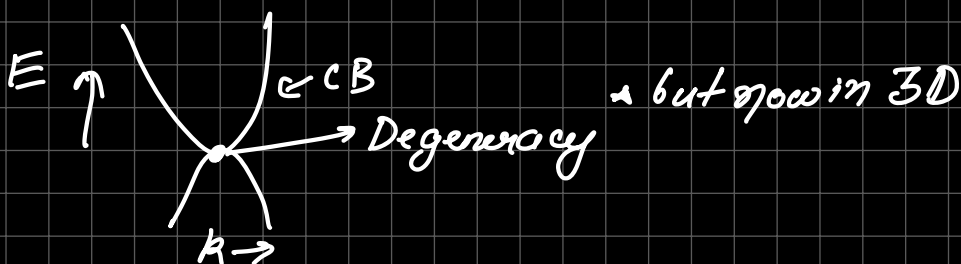
$$\partial_t (n_R - n_L) \neq 0$$

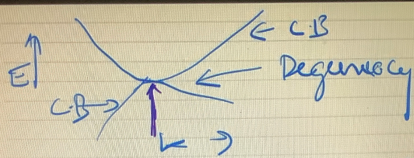


Smoking Gun confirmation
 "Chiral anomaly"

$\rightarrow$ ARPES :- Bandstructure

Cond mat realization of Weyl Hamiltonian.





Time reversal symmetry
& Inversion symmetry
can have implications
for degeneracy of
bands

What are conditions for
any generic two band

→ Hamiltonian to touch

$$H_0(k) =$$

What are conditions for
any generic two band

Hamiltonian to touch

$$H(k) = f_0(k)\mathbb{I} + f_1(k)\sigma_x + f_2(k)\sigma_y + f_3(k)\sigma_z$$

nothing to do
with spin just

two bands

To bring two bands into
one has to set

$$H(k) = f_0(k)\mathbb{I} + f_1(k)\sigma_x + f_2(k)\sigma_y + f_3(k)\sigma_z$$

[]

nothing to do
with spin just

two bands

To bring two bands into
one has to get
Coincidence. $f_1 = f_2 = f_3 = 0$

One can realize a WSM
Pugh

→ If at certain points in k -space, then
we "get" a way cone.

non-degenerate bands and
breaking symmetries

Now we write Hamiltonian
in the effective low-energy
form

$$H(k) \sim f_0(k_0)\mathbb{I} + v_0 \cdot \delta k \mathbb{I} + \sum_{\alpha=1,2} v_\alpha \cdot \delta k \sigma_\alpha$$

in-general in C.M.
realizations of WSM

→ $\frac{\partial E}{\partial k}$ doesn't have
to be
same
in all 3 directions

→ Lorentz invariance may not be
obtained

why do Weyl nodes
occur in pairs

$C=+1$ $C=-1$

Chirality $C = \text{sgn}(v_x \cdot v_y \times v_z)$
integrated
over B.Z. Berry curvature

maintained.

$\propto B \cdot \vec{v}$

why do Weyl nodes
occur in pairs

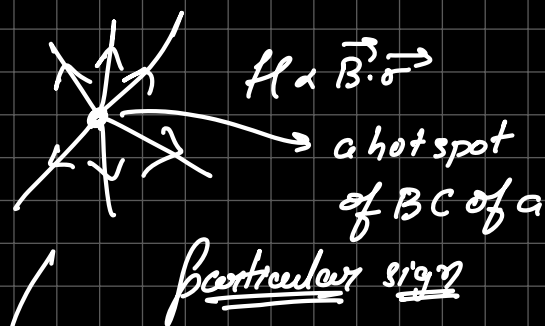
$C=+1$ $C=-1$

Chirality $C = \text{sgn}(v_x \cdot v_y \times v_z)$
integrated
over B.Z. Berry curvature

Another node $\rightarrow$ existence

$\downarrow$
IR + TRS
Conserved BC = 0

"physically justified way to
argue for it is net 0 sum
of B.C."



$\rightarrow$ If $C=0$, then ∇ another
hotspot of opposite sense has to
be here.

Two weyl nodes are sitting
on top of each other.

$\hookrightarrow$ "Dirac semi-metals"

nodes $\rightarrow$ Chiral anomaly &
other stuff doesn't
happen.

$\Rightarrow$ weyl/nodes = must occur in "pairs"

$\hookrightarrow$ simple argument $\Rightarrow \int \underline{BC} d^2z = 0$

Lattice models of Weyl fermions

chirality.

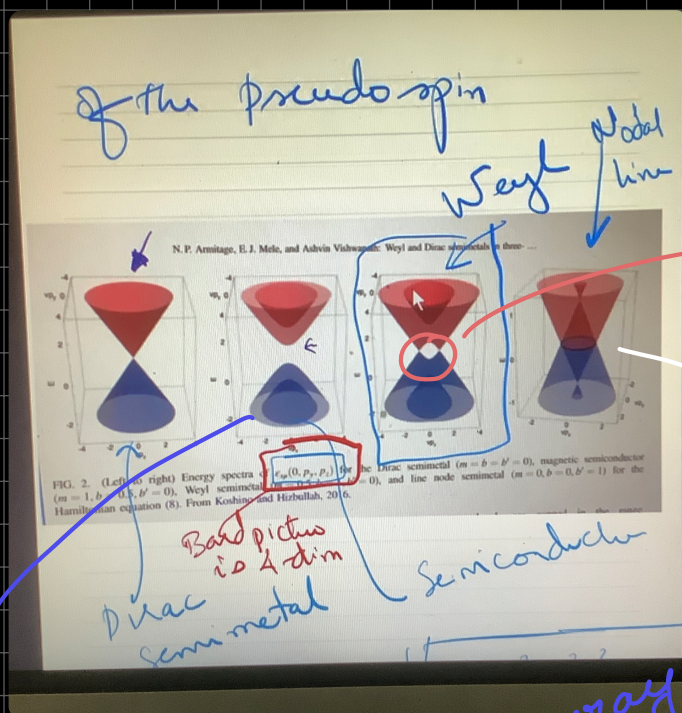
In general

$$H = v_z \sigma_z (\mathbf{G} \cdot \mathbf{r}) + m \sigma_z + b \sigma_z^2 + b' \sigma_z \sigma_x$$

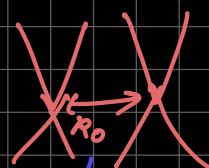
$$= \begin{pmatrix} m \sigma_z + b \sigma_z^2 + b' \sigma_x & v \mathbf{G} \cdot \mathbf{r} \\ v \mathbf{G} \cdot \mathbf{r} & -m \sigma_z + b \sigma_z^2 - b' \sigma_x \end{pmatrix}$$

here σ_i are pauli matrices

as b, b' are varied, $\rightarrow$ mimics all kinds of Band str.
 at some point $\exists$ weyl/nodes



@ Armitage's paper

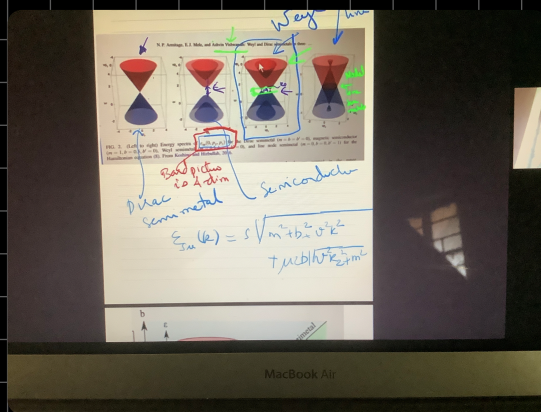


weyl Hamiltonian

touching point is a circle/loop

projected surface shown here

"nodal line semimetals"



Bond touching
are extremely
common

→ what en. window is it a linear approxⁿ
good?

touching shouldn't be
lowered under the
fermi energy